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AUTHOR Brazier, Gerald D.; Mick, Harold W.
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ABSTRACT

A model for learning and teaching mathematics is described which focuses on the relationships between mathematical concepts, the psychology of the learner, and instruction. The model distinguishes between figural and operational concepts and the types of abstraction (simple vs. reflective) needed to comprehend these concepts, that is, to assimilate them to appropriate schemas. Three learning-instructional phases are defined. (MN)

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A Logical-Cognitive View of Learning and Teaching Mathematics

Gerald D. Brazier and Harold W. Mick

Virginia Polytechnic Institute and State University

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The work that Harold and I have been doing has been described by some as a synthesis of Piaget, Skemp and Dienes. This is a characterization that I am certainly comfortable with and hope that I can explain here this morning. In some of the things we have written for the Models of Mathematics Learning Research Group,we begin with a discussion of Piaget, highlighting those aspects of his theory of cognitive development that we think are particularly relevant to mathematics learning. We then discuss a view of the structure of mathematics knowledge that can be seen as an application of Piaget's general theory of knowledge. Finally, with the help of the insights of Skemp and Dienes, we present some instructional consequences of our view of mathematics knowledge.

Today, rather than taking time to go over the hopefully familiar ground of Piagetian theory, I would like to plunge right into the discussion of mathematics concepts drawing upon the Piagetian sources in passing.

Mathematical Knowledge

Mathematical concepts can be thought of as being of two types: those which arise from objects and those which arise from actions. An abstraction process that abstracts what is shared in common by a class of objects has as its product what we call a figural concept. In the language of Piaget, such a process is simple abstraction. On the other hand, an abstraction process that abstracts what is shared

in common by a class of actions has as its product what we call an operational concept. Piaget would call this process reflective abstraction. Figural concepts are not meant to be limited to arising from physical objects that can be touched -- such things as polygons or polynomials are objects and can the bases of figural concepts. Likewise, operational concepts are not meant to be limited to actions on objects, such as the congruence of polygons, but can arise from actions on concepts or even conceptual structures.

These two types of concepts are then thought of as being brought together into conceptual structures or schemas. Figural concepts form the initial building blocks and these are linked by specialized operational concepts that permit skeletal formation of structures. Operational concepts can in turn be formed by attending to the actions between these skeletal structures and so create more organization. These collections of figural and operational concepts are the schemas referred to by Skemp (1971). They are both psychological and mathematical. Schemas are psychological in the sense that they are internal structures that are continually added to, modified, and consolidated; they are in themselves mental tools to apply to future learning tasks. Schemas are mathematical in the sense that they possess logical organization and sometimes form recognizable mathematical structures such as semi-groups, networks, etc. From both psychological and mathematical viewpoints there are endless ways in which figural concepts can be interrelated with the various operational concepts. The study of the ways in which these schemas are formed and function is assumed to be the very essence

of the psychology of learning mathematics (Skemp, 1971).

Piaget's operational structures of intelligence and these conceptual structures or schemas share certain similarities. From the standpoint of acquisition and form, both are internalized structures acquired by abstraction and self-regulated by equilibration, and both are mental constructs which serve as mental tools for further acquisition. There are also similarities from the standpoint of content or structure. Figural concepts relate to Piaget's physical knowledge (simple abstraction from the physical properties of objects). Weight, for example, is a physical characteristic of an object and thus would be classified by Piaget as physical knowledge. In our mathematical structures weight would be classified as a figural concept. The operational concepts are analogous to Piaget's logico-mathematical knowledge (reflective abstractions of coordinated actions which relate or transform objects or states). For instance, addition is considered to be an operation in both types of structures.

Operations are the essence of Piaget's cognitive structures and we believe that operational concepts play a dominant role within mathematical structures. In other words, abstracted coordinated actions play the major role in both systems of structures. For example, the essence of algebra is not the class of polynomial expressions, but the various ways these expressions are related (equations, inequalities, etc.) and acted upon (addition, multiplication, etc.).

Instruction

This brings us to a consideration of instruction. The aim of instruction is to bring learner and mathematics together in such a manner that understanding takes place. According to Piaget (1971, p. 15), "To know is to assimilate reality into systems of transformations." Skemp (1971) describes understanding this way: "To understand something means to assimilate it into an appropriate schema." Therefore, it would appear that the teacher's role in the instruction and learning processes is to prepare mathematics for abstraction into the learner's schemas and in a manner supportive of the learner's abstracting capabilities. The result should be schematic learning, that is, a building up of internal conceptual structures which consolidate within themselves and assist with future learning.

We assume that schemas follow the same basic formation patterns and abstraction processes as operational structures, so that Piaget's developmental psychology can serve as a guide for describing phases and processes associated with concept acquisition. Our current view posits three learning-instructional phases: exploration, assimilation, and formalization. Though suggestive of Piaget's three major stages we do not mean to imply that concept acquisition recapitulates general development. The more direct sources for these phases are Dienes (1961) and Steffe and Smock (1975).

In the brief time that remains, I would like to highlight some aspects of these phases. The purpose of the exploration phase is to begin the abstraction processes by presenting to the learner exemplars of the impending concept. The choice of exemplars in designing instructional

sequences depends to a great extent on the distinction between figural and operational concepts, since the learning processes associated with their acquisitions appear to be more unlike than alike. Part of their differences is attributable to the fact that figural concepts are internalized as images at what Piaget calls the representation level, whereas, operational concepts are internalized as operations at what Piaget calls the operational level (either concrete or formal operational). Therefore, qualitative variation of exemplars -- varying attributes, especially irrelevant ones and using non-exemplars as well as exemplars -- is important for figural concept acquisition. On the other hand, quantitative variation of exemplars -- repeated illustrations of the corresponding actions by varying only the numbers or figural concepts being acted upon -- is important for operation acted upon -- is important for operation acquisition. However, there is a very important qualitative variation associated with operation acquisition which uses exemplars to illustrate the associated reverse action, that is, exemplars vary between illustrating the "forward" action and the "backward" action. This is in keeping with Piaget's developmental psychology where he explicitly states that operations are by definition internalized actions capable of being reversed.

There doesn't seem to be any clearly defined end to the exploration phase. Loosely speaking, it is a time for play and organization and can be considered complete when the learner begins to form a rough idea of the concept under consideration. In the case of a figural concept some success is experienced in correctly classifying exemplars,

and in the case of operational concepts some success is experienced in performing the associated operation.

The assimilation phase spans the acquisition sequence from a rudimentary beginning of a concept to assimilation into an appropriate schema. In the case of operational concepts they are assimilated when they have been internalized as operations at which time we say that knowing or understanding has occurred. In the case of figural concepts, assimilation occurs when a representation has been achieved and integrated into an existing schema. Again, we say that knowing or understanding has occurred. Accomodation (probably in an unconscious form) will occur at this time to the extent that existing schemas may need to be slightly modified before assimilation can be achieved.

The formalization phase is characterized by reflective abstraction considered in the sense of being a mental process of reflection (at the level of thought a reorganization takes place). Skemp (1971) places important emphasis on the ability to reflect on our own mental processes which is generally lacking in learners before they become formal operational Instead of acting on reality we are acting on our cognitive structures and mental activities which requires this higher order mental process. According to Skemp (1971) when learners become able to reflect on their own schemas important further steps can be taken. Schemas can be communicated, old schemas can be replaced with new ones, and errors can be corrected in existing schemas. In general a type of consolidation takes place through strengthening existing bonds and by establishing further bonds with either existing or to be acquired schemas. The emphasis is on modifying (conscious acommodation) and establishing

relationships between classes within the schematic structure. The best ways to discuss the application of this second order of abstraction is in terms of integration and generalization. It appears as if time will not permit such a discussion this morning. Soon a monograph will be available in which our work will be presented in more detail along with the work of others in the models group.

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